

## nag\_bessel\_y0 (s17acc)

### 1. Purpose

**nag\_bessel\_y0 (s17acc)** returns the value of the Bessel function  $Y_0(x)$ .

### 2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_bessel_y0(double x, NagError *fail)
```

### 3. Description

The function evaluates the Bessel function of the second kind,  $Y_0$ ,  $x > 0$ .

The approximation is based on Chebyshev expansions.

For  $x$  near zero,  $Y_0(x) \simeq (2/\pi)(\ln(x/2) + \gamma)$ , where  $\gamma$  denotes Euler's constant. This approximation is used when  $x$  is sufficiently small for the result to be correct to **machine precision**.

For very large  $x$ , it becomes impossible to provide results with any reasonable accuracy (see Section 6.1), hence the function fails. Such arguments contain insufficient information to determine the phase of oscillation of  $Y_0(x)$ ; only the amplitude,  $\sqrt{2/x}$ , can be determined and this is returned. The range for which this occurs is roughly related to the **machine precision**: the function will fail if  $x \gtrsim 1/\text{machine precision}$ .

### 4. Parameters

**x**

Input: the argument  $x$  of the function.

Constraint:  $x > 0.0$ .

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

### 5. Error Indications and Warnings

#### NE\_REAL\_ARG\_GT

On entry,  $x$  must not be greater than  $\langle\text{value}\rangle$ :  $x = \langle\text{value}\rangle$ .

$x$  is too large, the function returns the amplitude of the  $Y_0$  oscillation,  $\sqrt{2/\pi x}$ .

#### NE\_REAL\_ARG\_LE

On entry,  $x$  must not be less than or equal to 0.0:  $x = \langle\text{value}\rangle$ .

$Y_0$  is undefined, the function returns zero.

### 6. Further Comments

#### 6.1. Accuracy

Let  $\delta$  be the relative error in the argument and  $E$  be the absolute error in the result. (Since  $Y_0(x)$  oscillates about zero, absolute error and not relative error is significant, except for very small  $x$ .)

If  $\delta$  is somewhat larger than the machine representation error (e.g. if  $\delta$  is due to data errors etc.), then  $E$  and  $\delta$  are approximately related by  $E \simeq |xY_1(x)| > \delta$  (provided  $E$  is also within machine bounds).

However, if  $\delta$  is of the same order as the machine representation errors, then rounding errors could make  $E$  slightly larger than the above relation predicts.

For very small  $x$ , the errors are essentially independent of  $\delta$  and the function should provide relative accuracy bounded by the **machine precision**.

For very large  $x$ , the above relation ceases to apply. In this region,  $Y_0(x) \simeq \sqrt{2/\pi x} \sin(x - \pi/4)$ . The amplitude  $\sqrt{2/\pi x}$  can be calculated with reasonable accuracy for all  $x$ , but  $\sin(x - \pi/4)$  cannot. If  $x - \pi/4$  is written as  $2N\pi + \theta$  where  $N$  is an integer and  $0 \leq \theta < 2\pi$ , then  $\sin(x - \pi/4)$  is determined by  $\theta$  only. If  $x \gtrsim \delta^{-1}$ ,  $\theta$  cannot be determined with any accuracy at all. Thus if  $x$  is greater than, or of the order of the inverse of **machine precision**, it is impossible to calculate the phase of  $Y_0(x)$  and the function must fail.

## 6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9 p 358.  
 Clenshaw C W (1962) *Mathematical Tables, Chebyshev series for mathematical functions* National Physical Laboratory H.M.S.O. 5.

## 7. See Also

nag\_bessel\_y1 (s17adc)

## 8. Example

The following program reads values of the argument  $x$  from a file, evaluates the function at each value of  $x$  and prints the results.

### 8.1. Program Text

```
/* nag_bessel_y0(s17acc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlb.h>
#include <nags.h>

main()
{
    double x, y;

    /* Skip heading in data file */
    Vscanf("%*[^\n]");
    Vprintf("s17acc Example Program Results\n");
    Vprintf("      x          y\n");
    while (scanf("%lf", &x) != EOF)
    {
        y = s17acc(x, NAGERR_DEFAULT);
        Vprintf("%12.3e%12.3e\n", x, y);
    }
    exit(EXIT_SUCCESS);
}
```

### 8.2. Program Data

```
s17acc Example Program Data
      0.5
      1.0
      3.0
      6.0
      8.0
     10.0
    1000.0
```

### 8.3. Program Results

```
s17acc Example Program Results
      x          y
 5.000e-01  -4.445e-01
 1.000e+00   8.826e-02
 3.000e+00   3.769e-01
 6.000e+00  -2.882e-01
 8.000e+00   2.235e-01
 1.000e+01   5.567e-02
 1.000e+03   4.716e-03
```

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